Numerical Modeling of Induction Shrink-Fits in Monolithic Formulation

Pavel Karban[1], Vaclav Kotlan[1], Ivo Dolezel[2]
[1]University of West Bohemia, Faculty of Electrical Engineering, Univerzitni 26, 306 14 Plzen, Czech Republic
E-mail: {karban, vkotlan}@kte.zcu.cz
[2]Czech University of Technology, Faculty of Electrical Engineering, Technicka 2, 166 27 Praha 6, Czech Republic
E-mail: dolezel@fel.cvut.cz

Abstract — Induction heating-based assembly and disassembly of axi-symmetric shrink fits is modeled. The task represents a triply coupled evolutionary problem characterized by mutual interaction of electromagnetic field, temperature field, and field of thermoelastic displacements. Its numerical solution is performed by a fully adaptive higher-order finite element method in monolithic formulation, using a code developed by the authors. All nonlinearities of the system (magnetic permeability and temperature dependencies of all material parameters) are respected. The methodology is illustrated by a typical example whose results are discussed.

I. INTRODUCTION

Shrink fits are widely used in numerous modern industrial technologies (production of shrunk-on rings, tires of railway wheels, armature bandages in electrical machines, fixing of machine tools, etc.). They are often realized by local induction heating of one metal part with the aim to increase its internal dimensions, inserting there another metal part, and consequent cooling of the whole system.

Modeling of the above phenomena, however, is still a complicated business, because the problem of both assembly and disassembly is evolutionary, includes interactions among three nonlinear physical fields (electromagnetic field, temperature field, and field of thermoelastic displacements), and, in specific cases, also the contact task.

The papers in the domain are rare. The authors solved some problems of induction heating-based thermoelasticity in [1] and [2] in the quasi-coupled formulations, with several simplifying assumptions. But the solution presented in this paper is already on a qualitatively much higher level. The model respecting all non-linearities of the system is solved in a monolithic formulation by own code based on a fully adaptive higher-order finite element method.

II. FORMULATION OF THE TECHNICAL PROBLEM

The investigated shrink fit serves for fixing a drill in the chuck of a high-revolution drilling machine. The process of its assembly and disassembly can be seen in Fig. 1. The thermal dilatability of the chuck must be substantially higher than that of the drill.

During the process of assembly the chuck **2** is heated by the inductor **3** until its internal diameter exceeds the diameter of the drill **1**. The drill is then inserted into the bore and the system is cooled until we obtain a shrink fit.

The disassembly is realized by induction heating of the whole system that causes displacements in the chuck **2** greater than those in the drill **1**. In a short time the shrink fit is released and the drill can be drawn out of the bore.

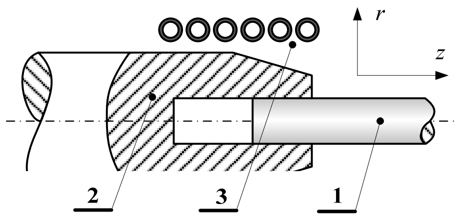

Fig. 1. Schematic arrangement of the system
1–drill, **2**–chuck, **3**–inductor

Suppose that the basic geometry of the chuck, drill, and torque to be transferred, are known. Now the task is

- to determine the necessary interference of the shrink fit, and
- to design the inductor (geometry and parameters of the field current) that satisfies all requirements concerning its assembly and disassembly.

III. CONTINUOUS MATHEMATICAL MODEL

Distribution of electromagnetic field in the system is described by the equation [3] for magnetic vector potential $\boldsymbol{A}$

$$\mathrm{curl}\left(\frac{1}{\mu}\mathrm{curl}\,\boldsymbol{A}\right)+\gamma\cdot\frac{\partial\boldsymbol{A}}{\partial t}=\boldsymbol{J}_{\mathrm{ext}}\,, \tag{1}$$

where μ denotes the magnetic permeability, γ the electric conductivity and $\boldsymbol{J}_{\mathrm{ext}}$ is the vector of the external harmonic current density in the inductor. Parameter γ is generally a function of the temperature T and μ a function of temperature T and magnetic flux density $\boldsymbol{B}$.

Distribution of the temperature field is described by the equation [4]

$$\mathrm{div}\left(\lambda\cdot\mathrm{grad}T\right)=\rho c\cdot\frac{\partial T}{\partial t}-w_{\mathrm{J}}\,,\quad w_{\mathrm{J}}=\gamma\cdot\left(\frac{\partial\boldsymbol{A}}{\partial t}\right)^{2} \tag{2}$$

where λ is the thermal conductivity, ρ denotes the mass density and c the specific heat. All these parameters are generally temperature-dependent functions. The internal sources only exist during the operation of the inductor.

The thermoelastic problem is described by the Lamé equation in the form [5]

$$\begin{aligned}&\left(\varphi+\psi\right)\cdot\mathrm{grad}\left(\mathrm{div}\,\boldsymbol{u}\right)+\psi\cdot\Delta\boldsymbol{u}-\\&-\left(3\varphi+2\psi\right)\cdot\alpha_{\mathrm{T}}\cdot\mathrm{grad}\,T+\boldsymbol{f}=\boldsymbol{0},\end{aligned} \tag{3}$$

where φ and ψ are coefficients given by the relations

$$\varphi=\frac{\nu\cdot E}{\left(1+\nu\right)\left(1-2\nu\right)},\ \psi=\frac{E}{2\cdot\left(1+\nu\right)}\,. \tag{4}$$

Here E denotes the modulus of elasticity and ν the Poisson coefficient of the contraction. Finally, $\boldsymbol{u} = \left(u_r, u_\varphi, u_z\right)$ represents the vector of the displacement, α_T the coefficient of the thermal dilatability, and $\boldsymbol{f}$ the vector of internal volumetric gravitational and Lorentz forces (in comparison with the thermoelastic stresses, however, these forces are very small and may be neglected).

IV. NUMERICAL SOLUTION

The numerical solution of the problem is realized by a fully adaptive *hp*-FEM. At each time level, optimal meshes are obtained automatically by independent adaptive processes. They dynamically change in time in accordance with the time evolution of results, respecting different features of particular fields. This is possible due to our own multi-mesh technique that allows us solving multiphysics problems monolithically, even though each physical field is discretized on a geometrically different mesh. This approach leads to a significant reduction of the size of the discrete problem and speeds up the whole computation.

Our own numerical SW Hermes and Agros [6] were used for the computation. They are capable of all the features mentioned above [7], such as the higher-order finite element method, automatic adaptivity on *hp*-meshes or assembling the monolithic stiffness matrix on geometrically different meshes.

V. ILLUSTRATIVE EXAMPLE

We present several results concerning the assembly of a shrink fit. The chuck made of a special tool steel is depicted in Fig. 2. The inductor formed by a hollow rectangular conductor intensively cooled by water carries harmonic current of density $J_\mathrm{ext} = 6\times10^7$ A/m^2 and frequency $f = 5$ kHz.

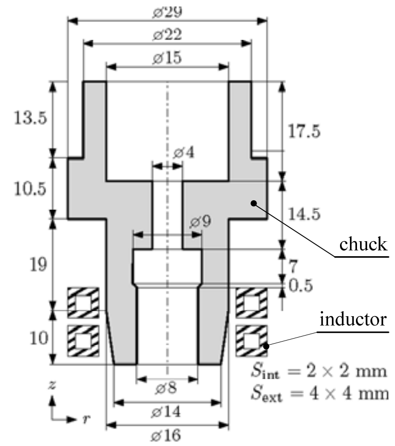

Fig. 2. Principal dimensions (in mm) of the chuck and inductor

In order to secure the torque that has to be transferred we used a shrink fit H7/s6 (hole/drill shank) with the maximum interference 19 μm (which represents the minimum necessary radial dilatation u_r of the hole). Figure 3 shows the radial dilatations of the diameter of the hole along its wall (measured from its bottom part, as shown in Fig. 2) in time. The time necessary to reach the required dilatation 19 μm along the whole wall (dashed line) is about 14 s.

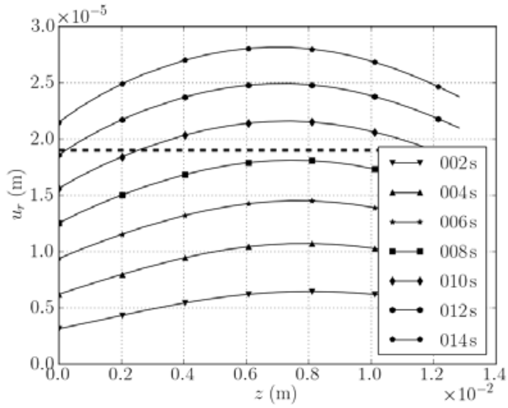

Fig. 3. Distribution of the displacements of the chuck hole in time

After inserting the cold drill shank (made of special steel for machine tools) into the hole the system starts to be cooled by fast flowing air (coefficient of convective heat transfer $\alpha = 150$ W/m^{2}K). In a short time the hole will shrink so that there will be a perfect contact between its wall and drill shank. Figure 4 depicts the time evolution of the total displacements of the internal surface of the hole at its particular points during both its heating and cooling.

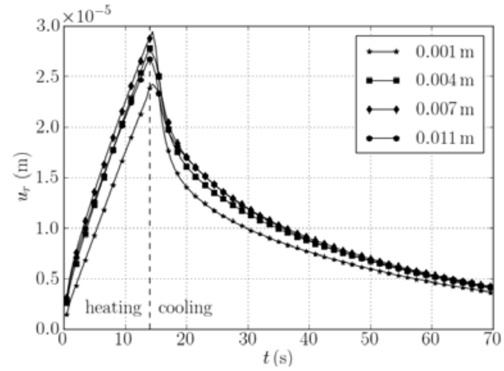

Fig. 4. Time evolution of the displacements at particular points of the chuck hole during the process of heating (up to 14 s) and cooling (later)

VI. ACKNOWLEDGMENT

The financial support of the Grant Agency of the Czech Republic (project No. P102/11/0498) and Grant Agency of the Academy of Sciences of the Czech Republic (project No. IAA100760702) is gratefully acknowledged.

VII. REFERENCES

[1] M. Skopek, B. Ulrych, I. Dolezel, "Optimized regime of induction heating of a disk before its pressing on shaft," *IEEE Trans. on Magn.*, vol.37, pp. 3380–3383, 2001.
[2] I. Dolezel, P. Karban, B. Ulrych, M. Pantelyat, Y. Matyukhin, P. Gontarowsky, and N. Shulzhenko, "Limit operation regimes of actuators working on principle of thermoelasticity," *IEEE Trans. on Magn.*, vol. 44, no. 6, pp. 810–813, 2008.
[3] J. A. Stratton, *Electromagnetic Theory*, McGraw Hill, NY, 2007.
[4] J. P. Holman, *Heat Transfer*, McGraw Hill, NY, 2002.
[5] B. Boley and J. Wiener, *Theory of Thermal Stresses*, NY, 1960.
[6] http://hpfem.org.
[7] P. Solin, J. Cerveny, L. Dubcova, and D. Andrs, "Monolithic Discretization of Linear Thermoelasticity Problems via Adaptive Multimesh *hp*-FEM," *J. Comput. Appl. Math*, vol. 234, pp. 2350–2357, 2010.